CALCOLO DEL DOMINIO DI RESISTENZA DI SEZIONI COMPOSTE ACCIAIO CLS SU BASE DELLE LEGGI COSTITUTIVE DEI MATERIALI – METODO E CASI APPLICATIVI REALI

CROSS-SECTION RESISTANCE OF COMPOSITE SECTIONS BASED ON A STRAIN LIMITED ANALYSIS – APPLICATION OF THE GENERAL METHOD TO REAL DESIGN CASES

Alessandro Angelini
STRUANG
Structural Engineering Solutions
39057 Appiano, Eppan (BZ) Italy
struang.engineering@gmail.com

Francesco Profico,
Riccardo Zanon
ArcelorMittal Global R&D
L-4221, Esch-sur-Alzette
Luxembourg
riccardo.zanon@arcelormittal.com

ABSTRACT

The use of steel-concrete composite cross-section is well established for beams and columns. For these, Eurocode 4 proposes a simplified design criteria based on plastic stress block stress state known as Rigid-Plastic method. The procedure is simple and efficient but has several limitations. There are cases where the geometric boundary conditions do not satisfy the validation assumptions for this method. High strength materials in accordance with the European design codes framework, are still not covered by the simplified Rigid-Plastic design methods of Eurocode 4. These have a larger elastic range and shorter plastic range leading to non-fully yielded ultimate stress state configurations. A similar case is the one involving slim-floor composite slabs with deep plastic neutral axis. In these configurations the use of Rigid-Plastic methods might lead to unprecise and unsafe results.

Eurocode 4 opens up the possibility of employing more advanced methods such as the Strain-Limited design method. Here the constitutive laws of materials are used. This method is well known in the scientific community but less used in professional practice. Despite the higher complexity it eliminates limitations of Rigid-Plastic analysis, it allows correct determination of ultimate limit state strength and ultimate deformation capacity. The availability of commercial validated Software on the market allows easy use of these methods and can facilitate their adoption.

SOMMARIO

L'utilizzo della sezione mista acciaio-calcestruzzo è comune sia per travi che per colonne. L'Eurocodice 4 propone un metodo di progettazione semplificato basato sulla resistenza plastica nell'ipotesi di distribuzione di sforzi di tipo stress-block. Questo è chiamato metodo Rigido-Plastico. La procedura è efficiente e di facile utilizzo ma presenta limitazioni. Vi sono casi pratici in cui le condizioni al contorno geometriche non soddisfano le ipotesi di validazione. I metodi di progettazione semplificati dell'Eurocodice 4 non sono ancora validati nel caso di materiali ad altissima resistenza previsti dalle Normative Europee. Questi presentano intrinsecamente un campo elastico più ampio ed una minore riserva plastica. Di conseguenza è difficile raggiungere una plasticizzazione completa della sezione. Un caso simile è rappresentato dai solai slim-floor. La posizione ribassata dell'asse neutro plastico non consente l'impiego del metodo semplificato. Il metodo Rigido-Plastico può quindi condurre a risultati errati e a sfavore di sicurezza.
L'Eurocode 4 apre alla possibilità di impiegare metodi più avanzati come il metodo Strain-Limitedin cui i legami costitutivi dei materiali sono implementati. Questo metodo è molto conosciuto nella comunità scientifica ma poco impiegato nella pratica professionale. Nonostante la più elevata complessità, elimina limitazioni dell'analisi Rigido-Plastica, consente una corretta determinazione della resistenza di stato limite ultimo e della capacità deformativa ultima. La disponibilità di Software commerciali validati sul mercato consente un facile impiego anche di questi metodi e può facilitarne l'adozione.

1 THEORY OF STRAIN BASED CROSS-SECTION ANALYSIS

For the ultimate limit state analysis of a cross-section, a condition close to collapse is analysed, assuming a non-linear relationship between stress and strain of both steel and concrete. It is therefore necessary to use algorithms which allows the cross-section to be analysed in such a condition. The software presented in this paper allows the analysis of a generic non-homogeneous cross-section subjected to biaxial bending.
When a structural element must resist to bending moment, the material, for homogenous cross-sections, or the materials, for non-homogenous one, must be capable to resists both tension and compression stress. The strain-based approach consists in calculating, for any defined fiber, the stress based on the strain value. For the most common structural solutions materials adopted are characterised by a non-linear stress-strain diagram as shown in the following figures.

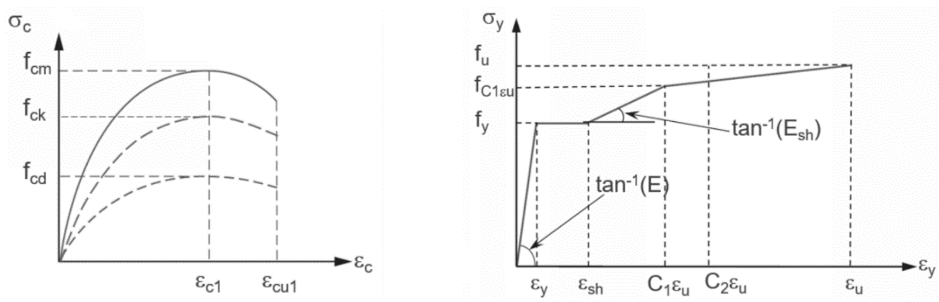

Fig. 1: On the left a typical stress-strain diagram of concrete for compression [1], on the right typical stress-strain diagram of steel for tension [4].

As for the concrete case, the relationship between the stress and the strain according to EN 1992 1.1 [2]. is defined by a specific non-linear law as follow.

$$\boldsymbol{\sigma_c = f_{cm} \cdot \left[\frac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta}\right]} \tag{1}$$

Where $\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}$ and $k = 1.05 \cdot E_{cm} \cdot |\varepsilon_{c1}|/f_{cm}$

An additional non-linear aspect is represented by the non-symmetrical behaviour between the compression and the tensile stress behaviour. The main assumptions [6] in the stress and strain calculation are:

- Knowledge of the stress-strain diagram for both fiber material and matrix material;
- The cross-sections remain plane in the deformed condition;
- Perfect bond between fiber material and matrix one, for RC section no sleep between concrete and rebars.

Latest assumption leads to assume an homogenisation coefficient defined by the following.

$$\boldsymbol{n = \frac{E_s}{E_{c_n}}} \tag{2}$$

Every time the axial force acts outside the cross-section centroid, it becomes subjected to a simple bending moment or biaxial-bending force. It is simple bending moment whether the eccentricity of the axial force is along one principal axis of the cross-section, otherwise it is a biaxial-bending.
Under this condition the cross-section change its configuration and the strain domain is represented by the following general law:

$$\boldsymbol{\varepsilon = \varepsilon_n + \varepsilon_f} \tag{3}$$

Where ε_n is the strain due to the axial load only while $\varepsilon_f = \chi \cdot y$ is the strain due to the bending moment, χ is the curvature and y is the distance of the fiber considered from the centroid of the cross-section. Therefore the strain values can be determined along the cross-section.

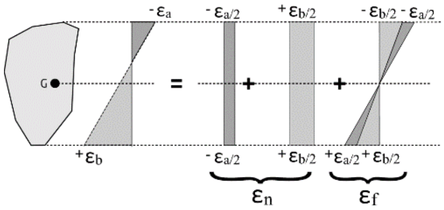

Fig. 2: Schematic subdivision of the strain diagram.

Then the problem presents two unknowns which, for the classical approach, are represented by the neutral axis depth and the strain on the compression side, while in the computational one are represented by ε_n and χ. The equations used to resolve this problem are represented by the equilibrium of the axial force and the equilibrium of the bending moment [5], as shown below.

$$\boldsymbol{N_{ext} = N_{int} = \int_A \sigma(\varepsilon) \cdot dA} \tag{4}$$

$$\boldsymbol{M_{ext} = M_{int} = \int_A y \cdot \sigma(\varepsilon) \cdot dA} \tag{5}$$

Due to the non-linear relationship between the stress and the strain of the material, the problem can be resolved with an iterative method. Then a specific calculation routine has been developed in order to subdivide a generic cross-section in several fibers. This allows the resolution of equation (4) and (5) through a simple algorithm which calculates the fiber stress based on the strain field which satisfy the equilibrium of axial forces and bending moments.

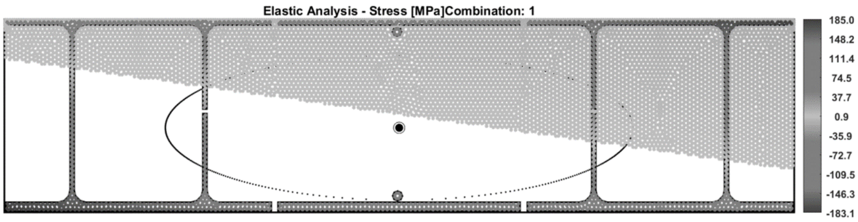

Fig. 3: Example of strain-based analysis of a composite cross-section, colormap of the tress values, MPa.

Based on the main strain relationship (3) and assuming the following initial values:

$$initialCurv = 10^{-7}$$
$$maxCurv = 10^{-3}$$
$$n = number\ of\ points$$
$$failure = 0$$
$$A = [initialCurv; \dots; maxCurve]$$
$$i = 0$$
$$tol = 0.01$$
$$nof = number\ of\ fibers$$

the moment curvature calculation can be defined using the following algorithm:

while *failure =0*
    $i = i + 1$
    $\theta_i = C(i)$
    $\delta = 10 \cdot tol$
    $\varepsilon_1 = 0.1$
    $\varepsilon_2 = -0.1$
    $Calculate\ \ \delta_1 = \delta(\varepsilon_1) = N_{ed} - F_1$
    $Calculate\ \ \delta_2 = \delta(\varepsilon_2) = N_{ed} - F_2$
    $\delta_x = \delta(\varepsilon_2)$
    while $\delta_x > tol$
        if $\delta_x \cdot \delta_1$ *<= 0*
            $\varepsilon_2 = x$
        else
            $\varepsilon_1 = x$
            $\delta_1 = \delta_x$
        End
        $x = \frac{\varepsilon_1 + \varepsilon_2}{2}$
        $Calculate\ \ \delta_x = \delta(x) = N_{ed} - F_x$
    End
    $\varepsilon_n = x$
    for $j = 1{:}\, nof$
        $calculate\ \varepsilon$
        $calculate\ \sigma_i(\varepsilon_n; \theta)$
        $calculate\ M_{Rd} = \sum \sigma_i \cdot area_i \cdot y_i$
        if δ > *failure limit*
            failure =1
        End
    End
End

Fig. 4: Algorithm example of the moment-curvature diagram calculation.

The moment-curvature diagram is obtained using the previous algorithm and leads to a solution after a certain number of iterations as shown in the following convergence example.

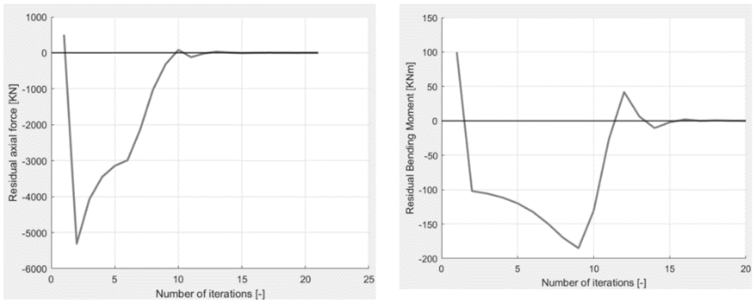

Fig. 5: Algorithm example of the moment-curvature diagram calculation.

Proposed software allowed the calculation of confined concrete which introduce an increase of both strain and design stress respect to the unconfined material, Fig. 6. According to EN 1992-1-1 [2] § 3.1.9 confined concrete strength and strain are obtained by the following formulas:

$$\boldsymbol{f_{ck,c} = f_{ck} \cdot (1.125 + 2.5\,\sigma_2/f_{ck})} \text{ for } \boldsymbol{\sigma_2 > 0.05 \cdot f_{ck}} \tag{6}$$

$$\boldsymbol{\varepsilon_{c2,c} = \varepsilon_{c2} \cdot (f_{ck,c}/f_{ck})^2} \tag{7}$$

The effective lateral compressive stress at ULS is defined is obtained with the following formula:

$$\boldsymbol{\sigma_2 = \alpha \cdot \sigma_l} \tag{8}$$

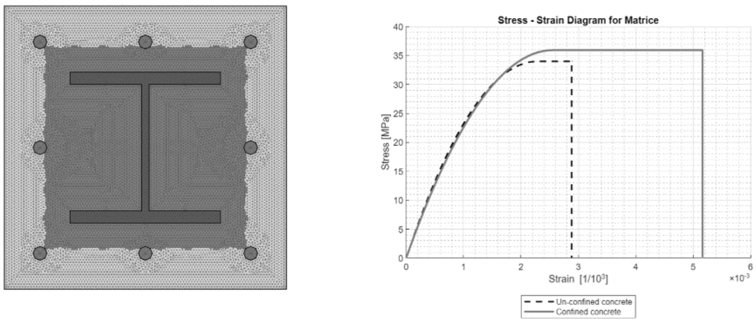

Fig. 6: Cross-section example of confined region considered (lest). Stress-strain diagram of confined and un-confined concrete.

The iterative process is carried out in control of curvature. By increasing the curvature value, at each iteration, the aim of the process is to define the normal strain ε_n which satisfy the equation (3). For the hypothesis of the plane section, the strain field is known and, according to the strain based approach, the stress domain is obtained using the non-linear stress-strain formulations of materials. The results are then represented in a moment-curvature diagram where the main events are represented, such us spalling, plasticization and failure of materials, Fig7.

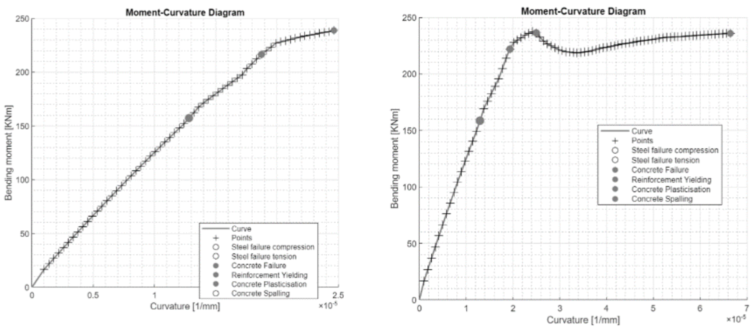

Fig. 7: Moment-curvature diagram for un-confined (left) and confined cross-section (right).

2 COMPARISON OF TWO STRAIN LIMITED ANALYSIS SOFTWARES

A comparison was made between the results of the strain limited analysis made using the "Biaxial-Crack" software [7], referred here as software I, and the respective results from the design calculation sheet for composite steel-concrete systems of PreCoBeam type, referred here as software II. This second software allows the strain limited analysis of one-symmetry axis sections using a fibre method. It is also possible to derive the Partial Shear Diagram of the composite section. The final purpose of the comparison is to validate the results of software II using software I. The materials considered are an S460 structural steel, a C35/45 concrete, a B500 type reinforcing steel. A pure bending action is considered. The resulting normal action on the section, obtained as an integral of the stress state, must therefore be zero. The analysis is done under Full Shear Interaction conditions (absence of slip strain between the concrete and the structural steel deformative state). The moment-curvature diagram of the section is obtained. The constitutive laws considered are a Parabola-Rectangle defined in accordance with EN1992-1-1 [2] and an elastic-plastic hardening for the definition of the structural steel and reinforcement. The discretization is made with fibres of a maximum thickness of 2.0 mm for software II.

Two separate composite sections are considered. The first, referred to here as Section 1, consists of a steel-concrete composite section with a single-T profile derived from a hot-rolled section type HE900AA. A constant thickness concrete slab of 110 mm is considered. The effective width of the slab is 5.00 m. The section is depicted in Fig. 8. The second section considered, referred to here as Section 2, consists of a slim-floor composite section with a PreCoBeam type shear connection. A single-T steel profile derived from a HE400M hot-rolled section is considered. The height of the concrete slab is 70mm and the effective width is 2.50m. The steel profile is partially encased in a concrete core. The section is depicted in Fig.8.

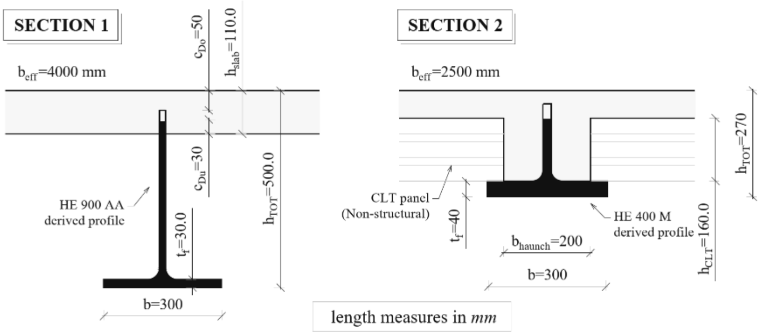

Fig. 8: Two composite steel-concrete sections considered for the strain limited analysis.

The results of the moment-curvature diagrams in pure bending in a Full Shear Interaction situation are illustrated for Section 1 and Section 2 in Fig.9 and Fig.10 respectively. For both sections, there is a good match of the results. This holds for both initial stiffnesses, Ultimate Limit State resistances and ultimate curvatures. Notice that small discrepancies in the resulting charts can be addressed to the different discretization, incrementation step and tolerances required by the two different software. As a consequence of the good correspondence the software II can be considered to be validated for these two types of composite sections under pure bending.

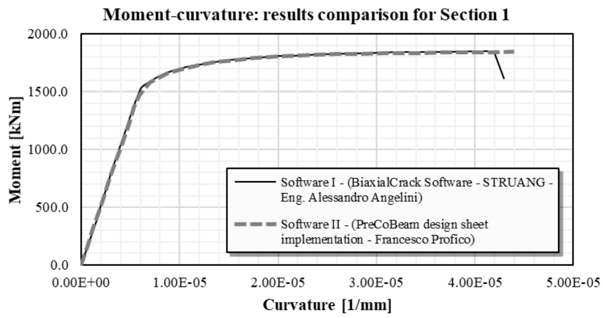

Fig. 9: Resulting moment-curvature diagram for Section 1 for pure bending in Full Shear Interaction.

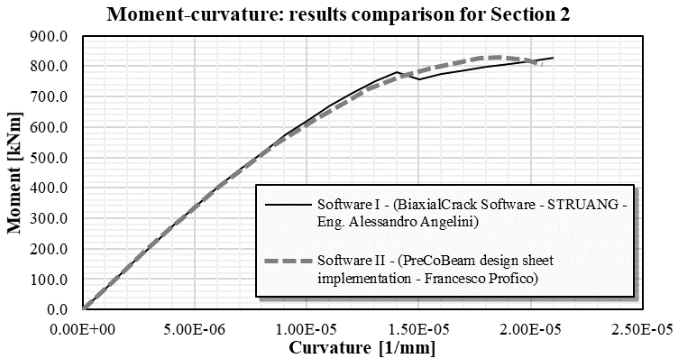

Fig. 10: Resulting moment-curvature diagram for Section 2 for pure bending in Full Shear Interaction.

8 CONCLUSIONS

The Strain Limited analysis method for the determination of the N-Mx-My resistance domain of generic steel-concrete composite sections is described. The method as well. This method has higher computational costs compared with the traditional Rigid-Plastic analysis allowed by Eurocode 4 [3] but allows the derivation of the moment-curvature response curve. The computational costs become compatible with the needs of engineering practice by a larger availability of validated software's. The Strain Limited method is more accurate (and therefore often needed) when using high-performance materials. In the present article the concept was demonstrated for the case of a specific example (Section 2). For instance, in the case of a very slender composite Slim-Floor slabs the steel profile behaves in a substantially elastic state since the plastic neutral axis is very deep. The Rigid-Plastic analysis would not have captured the correct Ultimate Limit State resistance of the element in this case. The other example showed instead the two analyses would have led to the same result in terms of ultimate bending resistance. As an additional favourable aspect, the Strain Limited method allows the determination of the ultimate curvature of the section and the initial stiffness that is not possible for traditional Rigid-Plastic methods.

REFERENCES

[1] Sargin, M. Stress-strain Relationship for Concrete and the Analysis of Structural Concrete Sections. Solid Mechanics Division University of Waterloo, (1971).
[2] EN1992-1-1 (2004): Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings, CEN European committee for standardization.
[3] EN1994-1-1 (2004): Eurocode 4: Design of composite steel and concrete structures - Part 1-1: General rules and rules for buildings, CEN European committee for standardization.
[4] Zhang, Q., Moment and longitudinal resistance for composite beams based on strain limited design method, PhD Thesis Université de Luxembourg (2020)
[5] Belluzzi, O. Scienza delle costruzioni, (1966)
[6] Toniolo, G.D., Di Prisco, M. Cemento armato, (2009)
[7] Angelini, A. STRUANG Software Manual, (2021)

KEYWORDS

Strain based analysis, strain limited design, cross-section resistance